①

$A \xleftrightarrow{90\,km} C \xleftrightarrow{150} B$, $240\,km$

רכבת של 7^{00} מ-A - מהירות X (משא)

מהירות רכבת של 8^{00} מ-C - $X+20$ (נוסעים)

משא $t_1 = \frac{240}{x}$

נוסעים $t_2 = \frac{150}{20+x}$

$t_1 - t_2 > 1\frac{1}{2}$ שעות

הפרש זמנים של 1 שעה ביציאה שעה וחצי בהגעה ויותר

$-1\frac{1}{2} > 0 \quad / \cdot x(20+x)$

$$\frac{240}{x} - \frac{150}{x+20} > 1\frac{1}{2} \quad / \cdot x(x+20)$$

$$240(x+20) - 150x - 1\frac{1}{2}x(x+20) > 0$$

$$240x + 4800 - 150x - 1\frac{1}{2}x^2 - 30x > 0$$

$$-1\frac{1}{2}x^2 + 60x + 4800 > 0 \quad / : -1\frac{1}{2}$$

$$x^2 - 40x - 3200 < 0$$

$$x_{1,2} = \frac{40 \pm 120}{2} \quad x_1 = 80, \quad x_2 = -40 \Rightarrow$$

$-40 \quad 80$

$$-40 < x < 80$$

וגם $0 < x$ $\Rightarrow$ $\boxed{0 < x < 80}$ א.

006

ב. $x = 80$ דמי

$$t_2 = \frac{150}{x+20} = \frac{150}{80+20} = \frac{150}{100} = 1.5 \text{ שעות}$$

$$S_{\text{דרך}} = x \cdot t = 80 \cdot 2.5 = \boxed{200}$$ דמ' מ-A

ז.

$a_n = 6^n + (-1)^{n+1}$

א. נניח שהטענה נכונה עבור $n=k$
k טבעי

$$\frac{a_k}{7} = \underset{\text{שלם}}{I} = \frac{6^k + (-1)^{k+1}}{7}$$

נבדוק עבור $n=1$

$$\frac{a_1}{7} = \frac{6^1 + (-1)^{1+1}}{7} = \frac{6+1}{7} = \frac{7}{7} = 1 \checkmark$$

נוכיח עבור $n=k$

$$a_{k+1} = 6^{k+1} + (-1)^{k+1+1}$$

$$= 6 \cdot 6^k + (-1) \cdot (-1)^{k+1}$$

$$\frac{a_{k+1}}{7} = \frac{6 \cdot 6^k - (-1)^{k+1}}{7} = 6\overbrace{\left[\frac{6^k + (-1)^{k+1}}{7}\right]}^{\text{שלם לפי הנחה}} - \frac{\cancel{7} \cdot (-1)^{k+1}}{\cancel{7}}$$

$$= 6I - (-1)^{k+1} = \underbrace{6I}_{\text{שלם לפי הנחה}} + (-1)^k = \text{שלם}$$

ח.

$|x+1| + |x| < 4$

I

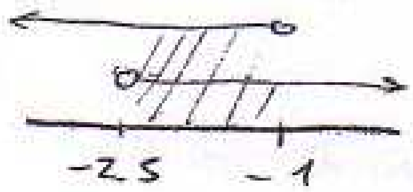
$x < -1$

אזי: $-(x+1) - x < 4$

$-2x - 1 < 4$

$-2x < 5 \quad | : -2$

$x > -2.5$



$-2.5 < x < -1$

II

$-1 \le x < 0$

אזי: $x + 1 - x < 4$

$1 < 4$

כל x בתחום

$-1 \le x < 0$

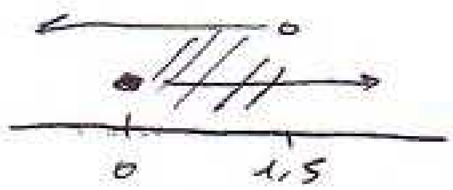
III

$0 \le x$

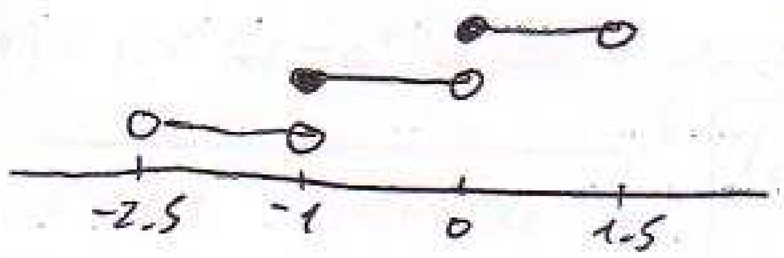
אם $x + 1 + x < 4$

$2x < 3$

$x < 1.5$



$0 \le x < 1.5$



$\Rightarrow$ $\boxed{-2.5 < x < 1.5}$

סופי.

006/3) $f(x)=\cos^2 x - a^2\cos x$ $\quad 0\le x\le 2\pi \quad a>\sqrt{2}$
$0\le x\le 360^\circ$

א. חיתוך ציר X: $Y=0$

$0=\cos^2 x - a^2\cos x$

$0=\cos x(\cos x - a^2)=0$

$\underline{\cos x=0}$ $\qquad \cos x = a^2 > 2$

I) $x=90^\circ+360^\circ k$ $\qquad \Downarrow$

II) $x=-90^\circ+360^\circ k$ $\qquad \phi$

$-1\le \cos x\le 1$

בתחום $X=90^\circ, 270^\circ$

$=\frac{\pi}{2}, \frac{3\pi}{2}$ $\quad \Rightarrow \boxed{(\frac{\pi}{2},0)\ (\frac{3\pi}{2},0)}$ א1

ב.

$f'(x)=-2\sin x\cos x + a^2\sin x = 0$

$\sin x(a^2-2\cos x)=0$

$\underline{\sin x=0}$ $\qquad \cos x = \frac{a^2}{2} > 1$

I) $x=0+360^\circ k$ $\qquad \Downarrow$

II) $x=180^\circ+360^\circ k$ $\qquad \phi$

$-1\le \cos x\le 1$

בתחום $X=0, 180^\circ, 360^\circ$

$=0, \pi, 2\pi$ $\quad \rightarrow \quad \underset{min}{(0,1-a^2)}\ \underset{max}{(\pi,1+a^2)}\ \underset{min}{(2\pi,1-a^2)}$

$f'(x)=-2\sin x\cos x + a^2\sin x$

$f'(\frac{\pi}{2})=-2\underbrace{\sin 90^\circ}\underbrace{\cos 90^\circ}_{=0}+a^2\underbrace{\sin 90^\circ}_{=1}=+a^2$

$f'(\frac{3\pi}{2})=-2\sin 270^\circ\underbrace{\cos 270^\circ}_{=0}+a^2\underbrace{\sin 270^\circ}_{=-1}=-a^2$

X	0 (קיצון)	$\frac{\pi}{2}$	π (קיצון)	$\frac{3\pi}{2}$	2π (קיצון)
f'	0	$+$	0	$-$	0
	min	$\nearrow$	max	$\searrow$	min

$f(0)=\cos^2(0)-a^2\cos(0)=1-a^2$

$f(\pi)=\cos^2(\pi)-a^2\cos(\pi)=(-1)^2-a^2\cdot(-1)=1+a^2$

$f(2\pi)=\cos^2 360^\circ - a^2\cos 360^\circ = 1-a^2$

א2

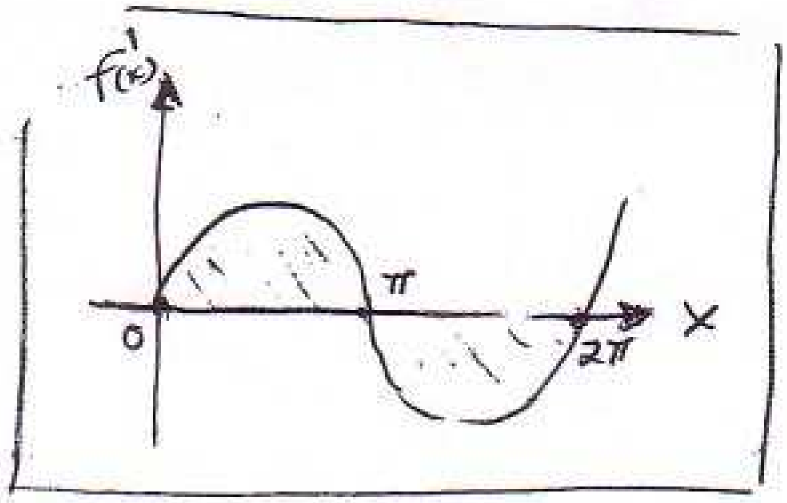
min מוחלט: $(0,1-a^2)\ (2\pi,1-a^2)$

max מוחלט: $(\pi,1+a^2)$

א3

$f'(x)$ חיובי: $0<x<\pi$

$f'(x)$ שלילי: $\pi<x<2\pi$



$f'(0) = f'(\pi) = f'(2\pi) = 0$

$$S = 16 = \int_0^{\pi} f'(x)dx + \left| \int_{\pi}^{2\pi} f'(x)dx \right| = f(x) \Big|_0^{\pi} + \left| f(x) \Big|_{\pi}^{2\pi} \right|$$

$$= \cos^2 x - a^2 \cos x \Big|_0^{\pi} + \left| (\cos^2 x - a^2 \cos x) \Big|_{\pi}^{2\pi} \right|$$

$$= (\cos^2\pi - a^2\cos\pi) - (\cos^2 0 - a^2 \cos 0)$$

$$+ \left| (\cos^2 2\pi - a^2 \cos 2\pi) - (\cos^2\pi - a^2\cos\pi) \right| =$$

$$= ((-1)^2 - a^2 \cdot (-1)) - (1^2 - a^2 \cdot 1)$$

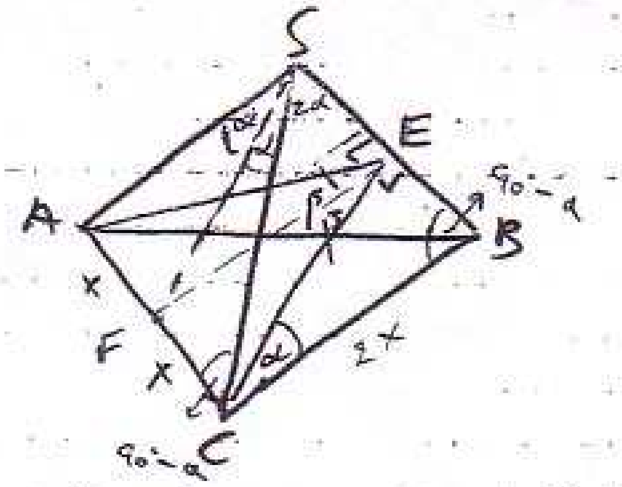
$$+ \left| (1^2 - a^2 \cdot 1) - ((-1)^2 - a^2 \cdot (-1)) \right| =$$

$$= 1 + a^2 - 1 + a^2 + \left| 1 - a^2 - (1 + a^2) \right| =$$

$$= 2a^2 + \left| 1 - a^2 - 1 - a^2 \right| = 2a^2 + \left| -2a^2 \right| =$$

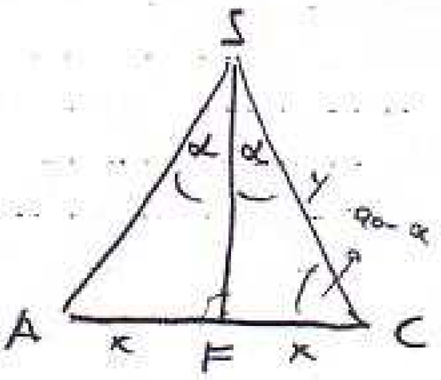
$$= 2a^2 + 2a^2 = 4a^2 = S = 16$$

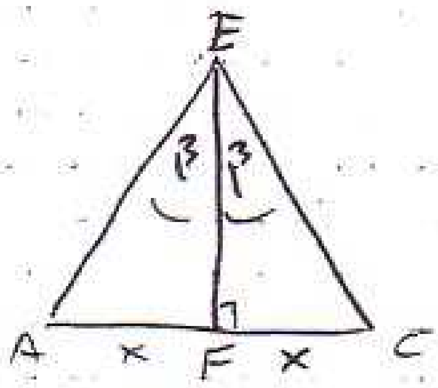
$$\Rightarrow a^2 = 4 \quad \Rightarrow \boxed{a = 2}$$



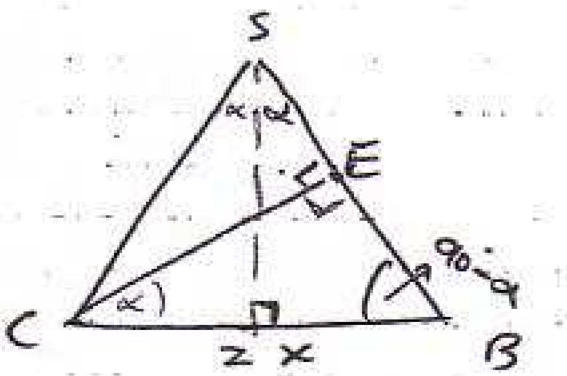
$$\sphericalangle ASB = \sphericalangle ASC = \sphericalangle BSC = 2\alpha$$

$$\sphericalangle AEC = 2\beta \qquad AC = 2X \text{ נתון}$$





$\underline{\Delta ACE}$: $\sin\beta = \dfrac{x}{CE}$



$\underline{\Delta SBC}$: $\cos\alpha = \dfrac{CE}{2x}$

$\Rightarrow CE = 2X\cos\alpha$

$$\Rightarrow \sin\beta = \frac{\cancel{x}}{2\cancel{x}\cos\alpha} = \boxed{\frac{1}{2\cos\alpha} = \sin\beta}$$

$\text{היקף } P_{ABC} = 3\cdot(2x) = 6X$

$\underline{\Delta ASC}$: $\dfrac{x}{SC} = \sin\alpha \Rightarrow SC = \dfrac{x}{\sin\alpha}$

$\text{היקף } P_{SAC} = 2\cdot SC + AC = \dfrac{2x}{\sin\alpha} + 2x$

לכן $P_{SAC} = P_{ABC} \Rightarrow \dfrac{2x}{\sin\alpha} + 2x = 6x \quad |:x$

$\dfrac{2}{\sin\alpha} + 2 = 6$

$\dfrac{2}{\sin\alpha} = 4 \quad \Rightarrow \quad 2 = 4\sin\alpha \quad |:4$

$\sin\alpha = \dfrac{1}{2} \Rightarrow \alpha = 30^\circ$

$\Rightarrow \cos\alpha = \cos 30^\circ = \dfrac{\sqrt{3}}{2}$

$$\sin\beta = \frac{1}{2\cos\alpha} = \frac{1}{2\cdot\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} \Rightarrow \boxed{\beta = 35.26^\circ}$$

$f'(x)=2x \qquad f'(t)=2t=m \qquad (t,t^2)$

משיק:

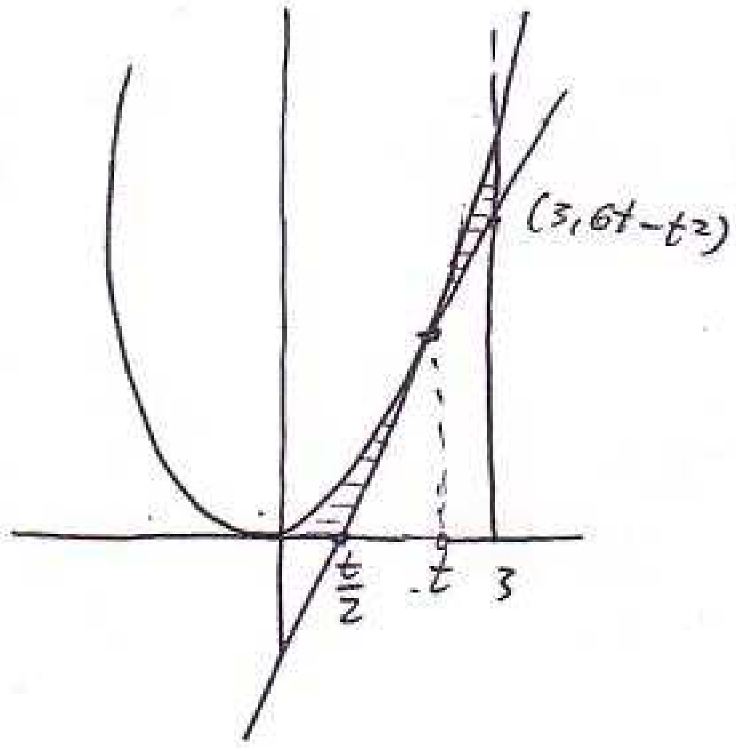
$y-t^2=2t(x-t)$

$y-t^2=2tx-2t^2 \Rightarrow y=2tx-t^2$

$0=2tx-t^2$

$t^2=2\cancel{t}x \quad |:t$

$t=2x \Rightarrow x=\frac{t}{2}$



$S=\int_0^3 x^2dx-\int_{\frac{t}{2}}^{3}(2tx-t^2)dx$

$=\frac{x^3}{3}\Big|_0^3-\left[\cancel{2}t\cdot\frac{x^2}{\cancel{2}}-t^2x\right]\Big|_{\frac{t}{2}}^{3}=$

$=\left(\frac{3^3}{3}-0\right)-\left[\left(t\cdot3^2-t^2\cdot3\right)-\left(t\cdot\left(\frac{t}{2}\right)^2-t^2\cdot\frac{t}{2}\right)\right]=$

$=9-\left[9t-3t^2-\left(\frac{t^3}{4}-\frac{t^3}{2}\right)\right]=9-\left[9t-3t^2-\left(-\frac{t^3}{4}\right)\right]$

$S(t)=-\frac{t^3}{4}+3t^2-9t+9$

$S'(t)=-\frac{3t^2}{4}+6t-9=0 \quad |\cdot4 \Rightarrow -3t^2+24t-36=0$

$t_{1,2}=\frac{-24\pm12}{-6} \to t=\frac{-36}{-6}=6 \quad$ לא $\quad t<3$

$\to t=\frac{-12}{-6}=2 \to \boxed{t=2}$

$S'(1)=\frac{-3\cdot1}{4}+6\cdot1-9=-$

$S'(3)=\frac{-3\cdot3^2}{4}+6\cdot3-9=+$

t	1	2	3
$S'(t)$	↘	0	↗
		min	

$t=2$:

משיק $y=2tx-t^2=4x-4$

$R=y(3)=4\cdot3-4=\boxed{8=R}$

$S(2)=-\frac{3\cdot2^3}{4}+3\cdot2^2-9\cdot2+9=1$

$\boxed{S_{min}=1}$